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Modulated inflation from kinetic term

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Abstract

We study modulated inflation from kinetic term. Using the Mukhanov-Sasaki variable, it is possible to determine how mixing induced by the kinetic term feeds the curvature perturbation with the isocurvature perturbation. We show explicitly that the analytic result obtained from the evolution of the Mukhanov-Sasaki variable is consistent with the δN-formula. From our results, we find analytic conditions for the modulated fluctuation and the non-Gaussianity parameter.

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1 Introduction

Since the mass of the Higgs field in the Standard Model (SM) is much smaller than the Planck scale, it is natural to expect that there is a mechanism (solution to the hierarchy problem) that causes the mass of the scalar fields to be much lighter than the Planck scale. In fact, string theory and supersymmetric models predict many light scalar fields whose expectation values determine the parameters of low-energy effective action. During inflation, light fields ($\mathcal{M}_i$) may lead to vacuum fluctuations that appear as classical random Gaussian inhomogeneities with an almost scale-free spectrum of amplitude $\delta\mathcal{M}_i$. Since the wavelength of the fluctuations is stretched during inflation over the Hubble horizon after inflation, the vacuum fluctuations of the light fields can be related in various ways to the cosmological curvature perturbation in the present Universe. In this paper, we consider modulated inflation as a mechanism relating the isocurvature perturbation to the curvature perturbation of the Universe. In this paper, we consider modulated inflation as one of the mechanisms that relate the isocurvature perturbation to the curvature perturbation of the Universe. The basic idea of modulated inflation is very simple. We first introduce modulated inflation mentioning the difference between modulated perturbation scenarios and multi-field (double) inflation.

We start with the conventional equation for the number of e-foldings elapsed during inflation:

$$N = \frac{1}{M_p^2}\int_{\phi_e}^{\phi_N} \frac{V}{V_\phi} d\phi, \tag{1.1}$$

where ϕ_N is the value of inflaton field ϕ corresponding to N e-foldings, and ϕ_e denotes the end-point of inflation. Using the δN-formula, we find that the fluctuation of the spectrum $\delta\phi_N = H_I/2\pi$ leads to the spectrum of the density perturbation

$$\delta_H^2 = \frac{4}{25}(\delta N)^2 = \frac{4}{25}\left(\frac{V}{M_p^2 V_\phi}\frac{H_I}{2\pi}\right)^2, \tag{1.2}$$

where M_p is the reduced Planck mass and H_I is the Hubble parameter during inflation. In addition to the standard inflation scenario in which $\delta\phi_N$ leads to the density perturbation, one may expect more generically other scalar fields may play a similar role. The first specific example of this has been given by Bernardeau et al.[1] for modulated couplings in hybrid-type inflation, in which ϕ_e depends on light fields through moduli-dependent

couplings. Lyth[1] considered a multi-inflaton model of hybrid inflation and found another realization of $\delta\phi_e$-induced curvature perturbation: "generating the curvature perturbation at the end of inflation". More recently, we considered trapping inflation combined with inhomogeneous preheating and found a different mechanism for generating the curvature perturbation at the end of weak inflation[2] caused by the fluctuation of the number density (δn) of the preheating field.

In addition to modulated scenarios related to the perturbation $\delta\phi_e$, we have recently proposed a new modulated scenario, modulated inflation[3]. In Eq.(1.1), fluctuations induced by the other components V_ϕ and M_p may generate curvature perturbation if the components are modulated during inflation. Based on this simple idea, we considered a new mechanism for generating the curvature perturbation[3] that relies on neither $\delta\phi_N$ nor $\delta\phi_e$. The source of the curvature perturbation in this scenario is the explicit $\mathcal{M}$-dependence of the inflaton velocity $\dot{\phi} \simeq V_\phi/3H_I$. The equation that relates the $\delta\dot{\phi}$-perturbation to the curvature perturbation is $N_\phi = H_I/\dot{\phi}$. The integration $N = \int_{\phi_e}^{\phi_N} N_\phi d\phi$ allows δN_ϕ to feed the curvature perturbation with the modulated perturbation continuously.[2] Another way to see the source in the new scenario is to consider the conventional evolution of the curvature perturbation

$$\dot{\mathcal{R}} = -H\frac{\delta P}{\rho + P}, \tag{1.3}$$

where the pressure perturbation $\delta P \simeq \dot{\phi}\delta\dot{\phi}$ is generated by the modulated perturbation.

On the other hand, in multi-field (double) inflation scenario[4], the change in the co-moving curvature perturbation caused by the isocurvature perturbation δs is given by

$$\dot{\mathcal{R}}|_s \simeq \frac{2H_I}{\dot{\sigma}}\dot{\theta}\delta s, \tag{1.4}$$

where σ denotes the adiabatic component and $\dot{\theta} \neq 0$ appears at the "bend" in the inflaton trajectory. This discriminates the source terms in modulated perturbation scenarios from multi-field (double) inflation. The term "modulated perturbation"[1, 3] has been used to distinguish different origins of the cosmological perturbation. It might be confusing, but in a modulated perturbation scenario the light field ($\mathcal{M}$) can be identified with an additional inflaton field.[3] In this case, there can be two different sources; "bend" and the

[2]See also Fig.1.

[3]For specific example, see Ref.[1, 2, 5].

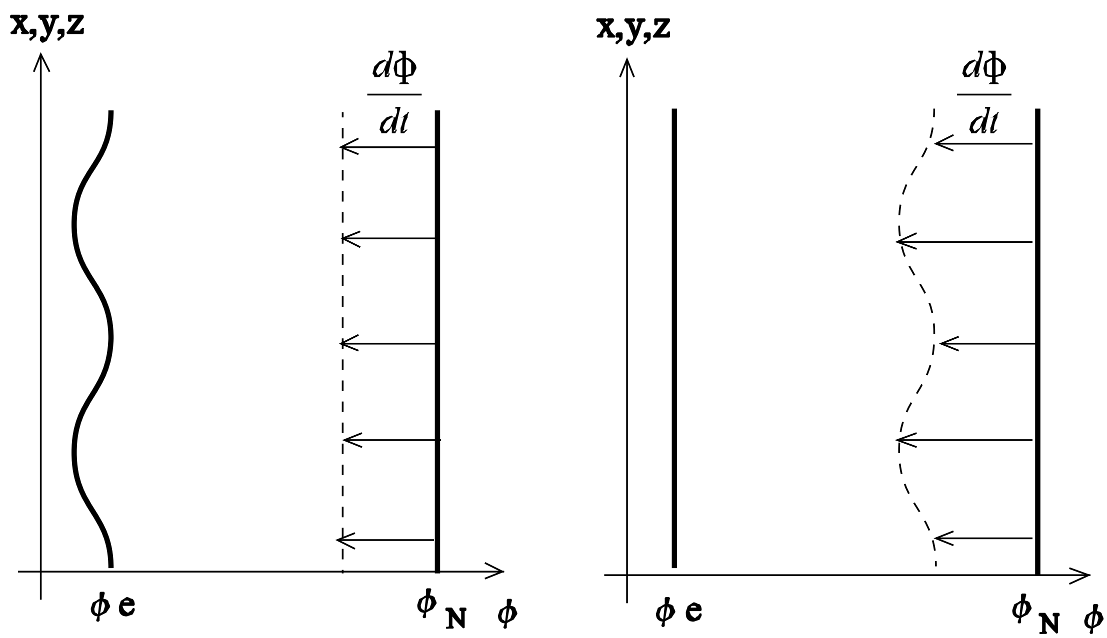

Figure 1: We consider the equation $\delta N = H\delta t_N$, where t_N denotes the time elapsed during inflation (from $\phi = \phi_N$ till the end $\phi = \phi_e$). In the left, the curvature perturbation is generated due to the modulated fluctuation $\delta\phi_e$ that leads to δt_N. In the right, the fluctuation is related to the modulated fluctuation $\delta\dot{\phi}$.

modulated perturbations. This approach is useful for brane inflation, since there can be several directions for the moving brane, as well as for the target brane.

As we mentioned above, there is a crucial difference between multi-field (double) inflation[4] and modulated perturbation scenarios[1, 3] in the mechanism that converts the isocurvature perturbation into curvature perturbation. However, the difference is not obvious when a lght field appears in the inflaton kinetic term. In section 2, we examine the differences and the similarities between the two scenarios when a lght field appears in the inflaton kinetic term.

2 Modulated inflation from kinetic term

We introduce a light field $\mathcal{M}$ and consider the kinetic term for the inflaton ϕ;

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\left(\nabla\mathcal{M}\right)^2 - \frac{1}{2}\omega(\mathcal{M})\left(\nabla\phi\right)^2 - W(\phi,\mathcal{M})\right], \tag{2.1}$$

where $\frac{\kappa^2}{8\pi} = G$ is the Newton's gravitational constant and $\omega(\mathcal{M})$ is a function of the moduli. The kinetic term may be given by a more general function, since any term that is not forbidden by symmetry may appear in the effective action. However, for the effective action during inflation and the kinetic term that can be approximated by a series expansion, we may disregard terms proportional to higher $(\nabla\phi)^n$. Of course, the above approximation is not always true, but we consider such action so that we can follow Ref.[8] in the following.[4]

We first consider a separable potential

$$W = V(\phi) + X(\mathcal{M}), \tag{2.2}$$

where $V(\phi)$ is the conventional inflaton potential. We consider flat potential for the light field $\mathcal{M}$. The coefficient of the kinetic term may be written as $\omega(\mathcal{M}) = 1 + \beta\frac{\mathcal{M}^2}{M_p^2}$ or $\omega(\mathcal{M}) = e^{b(\mathcal{M})}$.[5] These terms may appear in low-energy effective action. The definitions of the slow-roll parameters are

$$\begin{aligned}
\epsilon_{\mathcal{M}} &\equiv \frac{M_p^2}{2}\left(\frac{X'}{W}\right)^2 \\
\epsilon_{\phi} &\equiv \frac{M_p^2}{2\omega}\left(\frac{V'}{W}\right)^2, \\
\epsilon &\equiv \frac{\omega\dot{\phi}^2}{2M_p^2H_I^2} + \frac{\dot{\mathcal{M}}^2}{2M_p^2H_I^2},
\end{aligned} \tag{2.3}$$

where the prime denotes the derivative of the potential with respect to the corresponding field.

Note that we are not considering double inflation in which the isocurvature perturbation feeds the curvature perturbation mainly when there is a sharp bend in the trajectory. The bend occurs if the additional inflation stage is induced by the secondary inflaton field. Instead, we consider modulated inflation, in which modulated perturbation of the inflaton velocity sources the curvature perturbation. We first consider the evolution of the curvature perturbation paying attention to the source terms that can feed the curvature perturbation in different ways.

[4]See Ref.[6] for more specific examples. In Ref.[6], Ringeval et al. considered multi-field inflation in a brane model in which the specific form of the kinetic term is obtained from the string theory.

[5]Note that the former is identical to the latter when $b(\mathcal{M})$ is given by a logarithmic function.

2.1 Evolution of the curvature perturbation

Recently, cosmological perturbation in two-field inflation with a light field appearing in the inflaton kinetic term has been studied by Lalak et al.[8]. They calculated the spectra of curvature and isocurvature modes at the Hubble crossing and computed numerically the evolution of the curvature and isocurvature modes, showing how isocurvature perturbations significantly affect the curvature perturbation after Hubble crossing. Our first task is to obtain the analytic form of the curvature perturbation generated by the constant source term that can be related to the modulated inflation. We mainly follow Ref.[8] and consider the Mukhanov-Sasaki variable. At this time, we do not assume coincidence of the calculation given in Ref.[8] and the δN-formula, since the source term appeared in the δN-formula has been disregarded in the similar calculation when the inflaton has the standard kinetic term[4]. As will be shown later in the paper, the calculation is very simple in the δN-formalism. For convenience, we follow the notations in Ref.[8], with a slight difference. In Ref.[8], the light field $\mathcal{M}$ and the inflaton ϕ are denoted by ϕ and χ, respectively, and the coefficient $\omega(\mathcal{M})$ is given by an exponential $\omega = e^{b(\phi)}$. Since we are considering modulated inflation, the adiabatic component σ is essentially the same as the inflaton ϕ. Following the definitions in Ref.[8], we thus find for modulated inflation that the trajectory is given by

$$\begin{aligned} \cos\theta &\equiv \frac{\dot{\mathcal{M}}}{\dot{\sigma}} \simeq 0, \\ \sin\theta &\equiv \frac{\dot{\phi}\omega^{1/2}}{\dot{\sigma}} \simeq 1, \end{aligned} \tag{2.4}$$

where θ is a constant in modulated inflation, but changes abruptly when there is a sharp bend in double inflation. We do not consider the case when there is a sharp bend. The Mukhanov-Sasaki variable is defined by

$$Q_\sigma \equiv \delta\sigma + \frac{\dot{\sigma}}{H}\Phi, \tag{2.5}$$

where Φ is the metric perturbation. Using the slow-roll approximations, the equation of motion for the perturbation gives[8]

$$\begin{aligned} \dot{Q}_\sigma &\simeq AHQ_\sigma + BH\delta s \\ \dot{\delta s} &\simeq DH\delta s, \end{aligned} \tag{2.6}$$

where $\delta s = \delta \mathcal{M}$ for modulated inflation.[6] A, B, D are given by the following formula:

$$
\begin{aligned}
A &= -\eta_{\sigma\sigma} + 2\epsilon - \xi \cos\theta \sin^2\theta \simeq -\eta_{\sigma\sigma} + 2\epsilon \\
B &= -2\eta_{\sigma s} + 2\xi \sin^3\theta \simeq 2\frac{d\theta}{dN} - 2\xi \sin\theta \simeq -2\xi \\
D &= -\eta_{ss} + \xi \cos\theta(1+\sin^2\theta) \simeq -\eta_{ss} \simeq 0,
\end{aligned}
\tag{2.7}
$$

where ξ is defined by[7]

$$
\xi \equiv \frac{1}{\sqrt{2}}\frac{\omega'}{\omega} M_p \sqrt{\epsilon}. \tag{2.8}
$$

The source of the isocurvature perturbation feeding appears in Eq.(2.6) in the term proportional to B. In double inflation, there is a significant feeding with isocurvature perturbation if there is a sharp bend in the trajectory. The sharp bend leads to a large $d\theta/dN$, as has been discussed for double inflation[9]. In our scenario of modulated inflation, we do not expect such a bend in the trajectory. The main source in modulated inflation is the term proportional to ξ, which is small compared with $d\theta/dN$ at the bend, but gives a constant feeding and may become significant after integration. Because of the integration, the correction will be proportional to the number of e-foldings. Disregarding the first term in Eq.(2.6), we find

$$
\Delta Q \sim \int BH\delta s dt \sim B\delta s \int H dt \sim -2\xi N \delta s, \tag{2.9}
$$

which gives a simple result for the co-moving curvature perturbation:

$$
\Delta \mathcal{R} \equiv \frac{H}{\dot\sigma}\Delta Q_\sigma \sim -\frac{H}{\dot\sigma}\left[\sqrt{2}\frac{\omega'}{\omega} M_p \sqrt{\epsilon}\delta s N\right] \simeq -sign(\dot\sigma)\frac{\omega'}{\omega} N \delta s. \tag{2.10}
$$

These approximations are valid only if the time-dependence of the slow-roll parameters is small between Hubble crossing and the end of inflation. In the next subsection, we examine the slow-roll conditions used in the calculation[8].

Note that an inflation model with vanishing Q_σ at the horizon crossing does not cause a serious problem in modulated inflation scenario. Modulated inflation involves continuous

[6]Note that the constant source term proportional to $H\delta s$ has been disregarded in Ref.[4]. Therefore, the absence of this term discriminates between modulated inflation[3] and multi-field inflation for standard kinetic terms, as discussed in the introduction. On the other hand, the term proportional to $H\delta s$ appears in the similar calculation when there is a light field appearing in the inflaton kinetic term[8]. In this paper we examine the validity of this approach and the meaning of the term comparing with the δN-formalism.

[7]For the definitions of the η-parameters, see Ref.[8].

feeding with isocurvature perturbation, raising the curvature perturbation after horizon crossing. The feeding mechanism operates even if the standard inflaton perturbation is very small at the horizon crossing.

2.2 δN-formalism

Our second task is to find analytic result for the curvature perturbation following the δN-formalism. Variation of the action leads to the equations[10]

$$\begin{aligned}\ddot{\phi}+3H_I\dot{\phi}+\frac{V'}{\omega}+\frac{\omega'}{\omega}\dot{\phi}\dot{\mathcal{M}} &= 0\\ \ddot{\mathcal{M}}+3H_I\dot{\mathcal{M}}+X'-\frac{1}{2}\omega'\dot{\phi}^2 &= 0.\end{aligned}\tag{2.11}$$

Following Ref.[8], we find the velocity for the slow-roll inflaton field to be

$$\dot{\phi}\simeq-\frac{V'}{3H_I\omega},\tag{2.12}$$

where the approximation is valid if $|\omega'\dot{\mathcal{M}}/(3H_I\omega)|\ll 1$. Because of the additional term proportional to $\dot{\phi}^2$, the slow-roll condition for the light field $\mathcal{M}$ is

$$\frac{M_p^2}{2}\left(\frac{X'-\omega'\dot{\phi}^2/2}{W}\right)^2\ll 1.\tag{2.13}$$

If the potential X for the field $\mathcal{M}$ is very flat ($X'\simeq 0$), the above condition leads to

$$\frac{|\omega'|}{\omega}\ll\frac{3\sqrt{2}}{M_p\epsilon_\phi}.\tag{2.14}$$

This is not the condition for the potential, but is needed to ensure the slow motion of the field $\mathcal{M}$ during inflation. Otherwise, we find the conventional slow-roll condition $\epsilon_{\mathcal{M}}\ll 1$ when the term proportional to $\dot{\phi}$ is negligible compared with X'.

In modulated inflation scenario, the analytic calculation is very simple in the δN-formalism[3]. We follow Ref.[3] and start with the following formula[13]

$$N_\phi\dot{\phi}=-H_I,\tag{2.15}$$

where the subscript denotes the derivative with respect to the corresponding field. Using Eq.(2.12), we find

$$N_\phi=-\frac{H_I}{\dot{\phi}}\simeq\frac{3H_I^2}{V'}\omega.\tag{2.16}$$

The meaning of this equation is clear. Considering $\phi(N)$ as the "time" during inflation, N_ϕ gives the rate of change in the number of e-foldings. If N_ϕ is perturbed by the modulated fluctuation ($\delta\omega \neq 0$), it leads to δN after the "time"-integration. If ω is a constant during inflation, we find

$$N \simeq \omega \int \frac{3H_I^2}{V'} d\phi. \tag{2.17}$$

The fluctuation of the light field $\delta\mathcal{M}$ thus gives

$$\delta N \simeq \frac{\omega'}{\omega} N \delta\mathcal{M}. \tag{2.18}$$

The δN-formula is a very powerful tool in calculating the curvature perturbation in this set-up, since we can obtain analytic result without paying special attention to the curvature-isocurvature mixing during inflation. The fact that the two different calculations lead to the identical result is an important finding, since for standard kinetic terms there was no such correpondence.

Finally, we examine the conditions related to the cosmic microwave background (CMB) spectra. The condition for the modulated perturbation to dominate the curvature perturbation is

$$\left|\frac{N\omega'\omega^{-1}}{\sqrt{1/2\epsilon}M_p^{-1}}\frac{\delta\mathcal{M}}{\delta\sigma}\right| \simeq \left|N\frac{\omega'}{\omega}M_p\epsilon^{1/2}\frac{\delta\mathcal{M}}{\delta\sigma}\right| > 1. \tag{2.19}$$

If the modulated perturbation dominates curvature perturbation, the non-Gaussianity parameter can be large.[8] Note that the origin of this parameter is different from the that obtained in double inflation with a sharp bend[14]. In this case the value of the non-Gaussianity parameter is given by[13, 12, 1]

$$-\frac{3}{5}f_{nl} = \frac{1}{2}\frac{N_{\mathcal{M}\mathcal{M}}}{N_{\mathcal{M}}^2} \simeq \frac{\omega''\omega - \omega'^2}{2(\omega')^2 N}. \tag{2.20}$$

Using these results, we will examine the curvature perturbation in specific models of inflation.

[8]Non-Gaussian metric fluctuations related to the fluctuation $\delta\phi_e$ have been discussed by Bernardeau et al. in Ref.[11], prior to the publication of Ref.[1]. Note that in modulated inflation the source of the curvature perturbation is not related to $\delta\phi_e$.

2.3 Modulated inflation with a scalar field coupled to gravity 1

In Ref.[3], we mentioned that the fluctuation of the Planck mass may lead to the generation of curvature perturbation. In fact, the moduli-dependent Planck mass $M_p(\mathcal{M})$ can be seen as generalized scalar-tensor theory (Einstein gravity with a non-minimally coupled massless scalar field $\mathcal{M}$), which leads to the $\mathcal{M}$-dependence of the kinetic term after conformal transformations. Using the above analyses, it is possible to show that the isocurvature perturbation related to the modulated Planck scale can feed the curvature perturbation after horizon crossing. After conformal transformation, the potential depends on the scalar field $\mathcal{M}$ if there is no artificial cancellation.

Let us first examine the conditions for modulated inflation in the model in which $\mathcal{M}$ is non-minimally coupled with the scalar curvature[7];

$$\mathcal{L}_g = \sqrt{-\hat{g}}\left[\frac{1}{2\kappa^2} - \frac{1}{2}\beta\hat{\mathcal{M}}^2\right] R. \tag{2.21}$$

The quantity in brackets is positive as far as $\beta \ll 1$ and $\hat{\mathcal{M}} \leq M_p$. Note that induced coupling to the Ricci scalar may arise from one-loop gravity corrections. After conformal transformation

$$g_{\mu\nu} = \Omega^2 \hat{g}_{\mu\nu} \tag{2.22}$$

with

$$\Omega^2 = 1 - \beta\kappa^2\mathcal{M}^2, \tag{2.23}$$

we find the action for the inflaton kinetic term with

$$\omega = \frac{1}{1 - \beta\kappa^2\mathcal{M}^2} \tag{2.24}$$

and

$$W = \omega^2 V(\phi), \tag{2.25}$$

where $\mathcal{M}$ is redefined by

$$\mathcal{M} \equiv \int \Omega^{-1}\sqrt{1 - (1-6\beta)\beta\kappa^2\hat{\mathcal{M}}^2} d\hat{\mathcal{M}}. \tag{2.26}$$

The conformal transformation leads to the effective potential

$$W(\mathcal{M}, \phi) \simeq V(\phi) + 6\beta H_I^2 \mathcal{M}^2, \tag{2.27}$$

where we assumed $\beta\kappa^2\mathcal{M}^2 \ll 1$ in the last approximation. The model has been studied by Tsujikawa et al. in Ref.[7] by numerical computation. However the parameter space where the modulated perturbation is significant was "ruled out", due to the fact that the modulated perturbation leads to the distortions in inflaton perturbation. In fact, in modulated inflation, we consider the situation where the modulated perturbation dominates the inflaton perturbation, which may be seen as a "distortion". However, the important point is that the modulated perturbation can lead to a **successful** generation of the CMB spectra if appropriate conditions are satisfied. Modulated inflation gives a good approximation when the field $\mathcal{M}$ satisfies the slow-roll condition. In this case, the approximation $D \simeq 0$ in Eq.(2.7) is good and there is no strong amplification or suppression of $\delta\mathcal{M}$ during inflation. Using Eq.(2.19), we find the condition for the modulated perturbation to dominate the curvature perturbation to be

$$\left|2N\beta\epsilon^{1/2}\frac{\mathcal{M}}{M_p}\right| > 1, \tag{2.28}$$

where $|\beta| \ll 1$ and $\delta\mathcal{M} \simeq \delta\phi$ are considered. If there is no fine-tuning between the slow-roll parameters, the spectral index suggests that $|\epsilon| \le |\beta| \sim O(10^{-2})$ or $|\beta| \le |\epsilon| \sim O(10^{-2})$. We thus conclude that the ratio is at most $N_{\mathcal{M}}/N_\phi \sim 0.1$ for $\mathcal{M} \sim M_p$. Note that the above condition is valid only when there is no amplification or suppression of the fluctuations $\delta\phi$ or $\delta\mathcal{M}$ during inflation. We next consider fast-roll inflation[15] with $\eta_{\phi\phi} \ge 1$. The fluctuation of ϕ does not produce classical perturbations when $\eta_{\phi\phi}$ is larger than unity. For the fast-roll (hybrid) inflation with quadratic potential $\sim \pm m^2\phi^2/2$, the velocity of the inflaton field is given by[15]

$$\dot{\phi} = -F(\omega)H_I\phi, \tag{2.29}$$

where the function $F(\omega)$ is defined by

$$F(\omega) \equiv \sqrt{\frac{9}{4} \mp \frac{m^2}{\omega H_I^2}} - \frac{3}{2}. \tag{2.30}$$

Using this result, we find the fluctuation of the number of e-foldings to be

$$\delta N \simeq -\frac{F'N}{F}\delta\omega, \tag{2.31}$$

where the prime denotes the derivative with respect to ω. Considering the term proportional to $\dot{\phi}^2$, the slow-roll condition for $\mathcal{M}$ is

$$\frac{\omega'}{\omega} \ll \frac{3\sqrt{2}}{M_p \epsilon}. \tag{2.32}$$

In this case the modulated perturbation is the main source of the curvature perturbation, and the non-Gaussianity parameter f_{nl} can be large.

2.4 Modulated inflation with a scalar field coupled to gravity 2

The theory with a light scalar field $\hat{\mathcal{M}}$ coupled to gravity can be given by the action

$$S = \int d^4x \sqrt{-g} \left[f(\hat{\mathcal{M}})R - g(\hat{\mathcal{M}}) \left(\nabla \hat{\mathcal{M}} \right)^2 - \frac{1}{2} \left(\nabla \phi \right)^2 - V(\phi) \right]. \tag{2.33}$$

Considering Jordan-Brans-Dicke theory[16], f and g are written as

$$f = \frac{\hat{\mathcal{M}}}{16\pi}, \quad g = \frac{\omega_{BD}}{16\pi \hat{\mathcal{M}}}, \tag{2.34}$$

where ω_{BD} is the so-called Brans-Dicke parameter.[9] After a conformal transformation, the action in the Einstein frame is given by the action with the inflaton kinetic term with

$$\omega(\mathcal{M}) = \exp\left(-\beta \kappa \mathcal{M}\right) \tag{2.35}$$

and the potential

$$W = \omega(\mathcal{M})^2 V(\phi). \tag{2.36}$$

Here the dimensionless constant β is given by

$$\beta \equiv \sqrt{\frac{2}{2\omega_{BD} + 3}} \ll 1, \tag{2.37}$$

and $\mathcal{M}$ is the field after conformal transformation. Using Eq.(2.19), and introducing a new dimensionless parameter

$$\alpha_\delta \equiv \frac{\delta \sigma}{\delta \mathcal{M}}, \tag{2.38}$$

we find the condition for modulated inflation:

$$\beta > \frac{\alpha_\delta}{N\sqrt{\epsilon}}. \tag{2.39}$$

[9]Note that since $\mathcal{M}$ couples to matter, it will participate to the gravitational sector. In this case, M_p would not be the value measured in a Cavendish type experiment. See Ref.[22] for recent study.

These conditions suggest that the modulated perturbation cannot dominate the curvature perturbation for conventional slow-roll inflation. On the other hand, modulated perturbation can be significant if the inflaton is fast-rollong and has the parameter $\eta_{\phi\phi} \geq 1$, as we discussed in the previous subsection.

3 Conclusions and discussions

In this paper, we have studied modulated inflation from kinetic term. We showed explicitly that the analytic result obtained from the evolution of the Mukhanov-Sasaki variable is consistent with the δN-formula. Using the simple formula obtained in this paper, we found an analytic condition for modulated inflation. We also found analytic formula for the non-Gaussianity parameter in modulated inflation.

In modulated inflation, the integral of the source term related to the perturbation of the inflaton velocity leads to the curvature perturbation. We consider modulated inflation to be an alternative to conventional inflation, in the sense that it saves inflation when the conventional inflaton perturbation fail to generate the CMB spectra. For example, observation of a large non-Gaussianity parameter (if confirmed) can be a problem for single-field inflation. According to Yadav et al[21], the WMAP 3-year data may not favor single field slow-roll inflation.[10] Note also that the string η-problem may prevent a successful standard single-field inflation scenario. There are many attempts in this direction. A curvaton[17] generates the curvature perturbation long after inflation and saves inflation when the inflaton perturbation does not lead to the successful generation of the cosmological perturbation. The observation of a low-energy gravitational effect in the Large Hadron Collider(LHC) may put a strong upper bound for the inflation scale, but the bound can be evaded in many ways[18, 19]. Modulated perturbation may lead to inhomogeneous preheating[2] or inhomogeneous reheating after inflation[20]. Note that inhomogeneous preheating can work with a low inflation scale. The string η-problem may be solved by one of these alternatives[5]. Note that modulated inflation is consistent with fast-roll inflation. Moreover, these models are consistent with a large non-Gaussianity.

[10] Non-gaussianity may be generated at preheating[23].

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